University of Mannheim
School of Social Sciences
Math Refresher for Political Science, Fall 2025
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Problem Set: Analysis II

1. Suppose the function f is defined for all $x \in [-1.5, 2.5]$ by $f(x) = x^5 - 5x^3$.

   (a) Determine for which values of x the value of the function is equal to zero.

   (b) Calculate f'(x) and find the extreme points of f. What is the maximum/the minimum of the function?

   (c) Does the function have inflection points?

2. Which of the following functions of x are convex? Which are concave?

   (a) $f(x) = (2x - 1)^6$
   (b) $f(x) = 5x + 7$
   (c) $f(x) = x^5$
   (d) $f(x) = \sqrt{1 + x^2}$
   (e) $f(x) = x^5$ for $x \geq 0$
   (f) $f(x) = 5x^2 - x^4$ for $x \geq 1$

3. Appeasement Problem (Ashworth and Bueno de Mesquita, 2006)

   Two states must divide some territory. There is a status quo division, but one state (call is D) is dissatisfied with the status quo. The other state (call is S) is satisfied with the status quo division. S gets one chance to try to appease D by offering it some of the disputed territory. Let x be the fraction of $S's$ territory that it offers. S is uncertain about how dissatisfied D is. S believes that D will accept an offer of x with probability $p(x) = x$. If D accepts the offer, then war is averted and S is left with $1 - x$ of its territory. If D rejects the offer, then there is a war. S believes that it will win a war with probability q. Thus, q can be thought of as $S's$ relative military strength. If S wins the war, then S keeps all of its territory. If S looses the war, it ends up with none of the disputed territory.

   Given all of this, $S's$ maximization problem is given by

$$\max_x (1 - x)x + q(1 - x)$$

(a) Find the solution to $S's$ maximization problem.

(b) How does the level of appeasement, x, changes with $S's$ perception of its military strength?

4. A government has to decide about the allocation of its budget. Let x denote the share of the budget used for military and y the share of the budget used for social expenditures. The government has to use of all its budget and has the following utility function:

$$u(x, y) = e^{2x} + e^{2y}$$

Solve the government's optimization problem.

5. Consider the function $f(x) = (x^2 + 2x)e^{-x}$.

(a) Determine for which values of x the value of the function is equal to zero.

(b) Calculate f'(x) and find the extreme points of f. What is the maximum/the minimum of the function?

(c) Does the function have inflection points?

(d) Sketch the function and specify whether it is convex/concave (in sections).

6. Solve the indefinite integrals:

(a) $\int \frac{1}{\sqrt{x}} dx$

(b) $\int e^{-4t} dt$

(c) $\int x\sqrt{x} dx$

(d) $\int \frac{1}{x} dx$

(e) $\int (2x^2 + x - 3) dx$

(f) $\int \frac{(x^4 + 1)^2}{x^3} dx$

7. Calculate $\int_0^2 (2x^2 + x - 3) dx$. Hint: Make a sketch of the function before.